

(DN) ON BACK OF PACKET

Name \_\_\_\_\_ Per \_\_\_\_\_

LO: I can define area, find area, and explain dissection and Cavalieri's Principle as they relate to area and volume.

(1) What do we mean by "3D" or "3-dimensional?"  
calculator

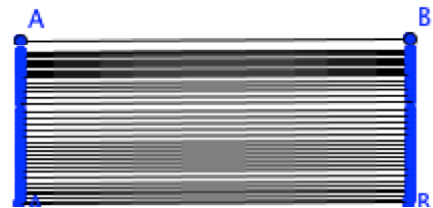
A point has no measure because a point represents a \_\_\_\_\_



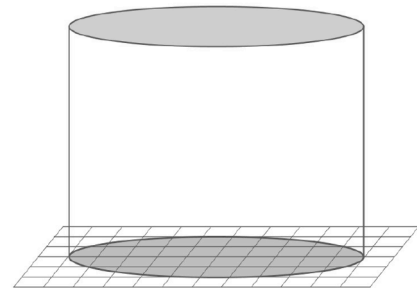
"Dragging" a point in one direction results in a \_\_\_\_\_ or \_\_\_\_\_ which has a measure of \_\_\_\_\_



"Dragging" a line across a surface results in a region which has a measure of \_\_\_\_\_



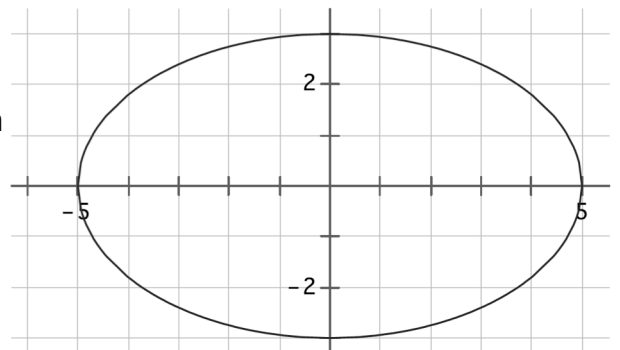
"Dragging" a region off of a plane results in a solid which has a measure of \_\_\_\_\_



(2) What is area?

(a) **Area** is a measurement that allows us to quantify 2-dimensional regions. The two dimensions can be seen in the diagram below. Units can be measured in the \_\_\_\_\_ direction and the \_\_\_\_\_ direction. The quantity of area of a square region bound by 1 x-unit and 1 y-unit is equal to 1 square unit.

(b) Refer to the figure on the graph at right. It takes one can of paint to cover a square unit in the coordinate plane. How many cans of paint are needed to paint the region within the curved figure?

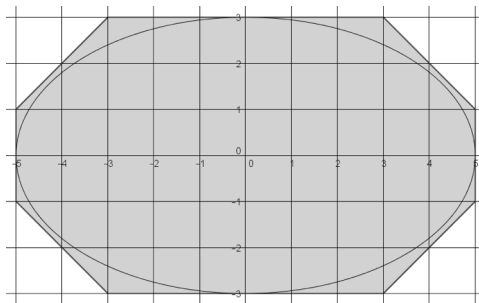
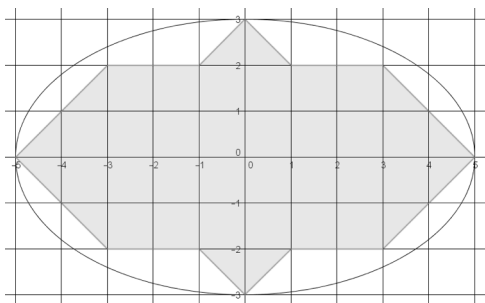


(3) How does the idea of dissection help us?

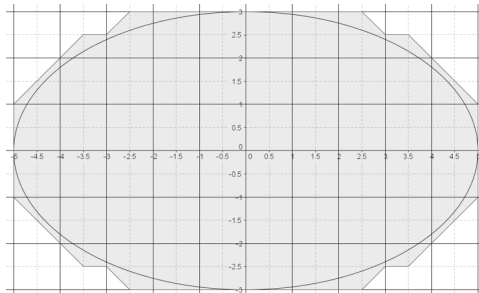
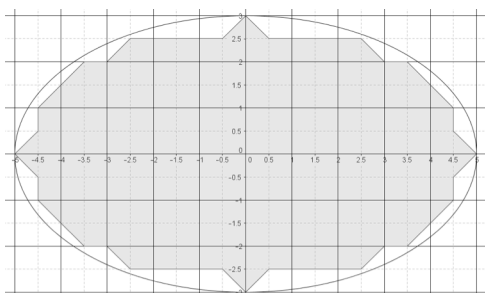
cont.  
calculator

(c) How do the pair of diagrams below help?

Make a conclusion about the area of the original shape based on these two shaded areas.



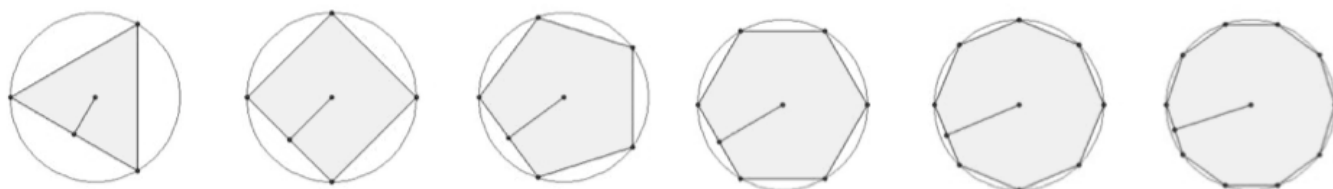
(d) How do the pair of diagrams below help us improve our conclusion from the last pair of diagrams?



(e) What if we continue using a smaller and smaller grid? Can we **squeeze** the area out of squares on a grid by reaching a limit?

(4) Where does the area formula for circles come from?

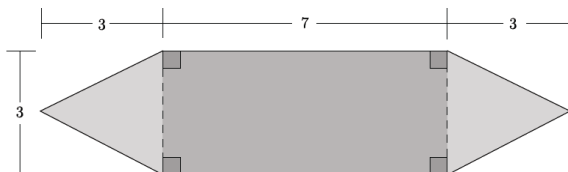
The circumference and area of a circle can also be found by reaching a limit. As the polygons have more sides, the perimeter of the polygon gets closer to the \_\_\_\_\_ of the circle and the area of the polygon gets closer to the \_\_\_\_\_ of the circle.



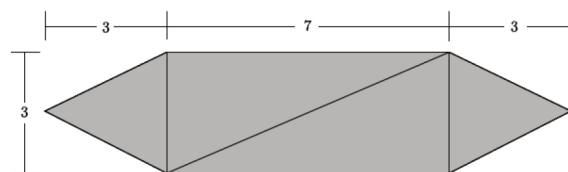
(5) **Dissection into triangles**

calculator

- (a) Dissect the shape below into 3 basic shapes and use the area formulas on your reference sheet (the back of 8.0) to calculate the area of the shaded region below.

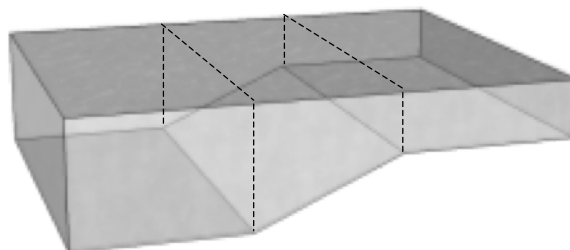
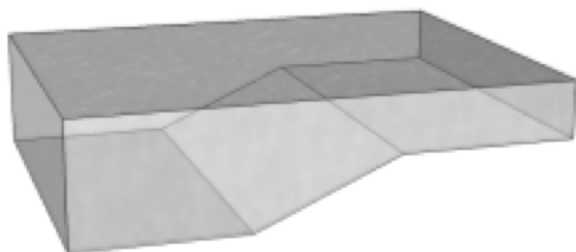


- (a) Will you get the same result by finding the areas of the triangles as shown below.



- (c) **Summary for area dissection:** The sum of the areas of all non-overlapping regions within a larger region is equal to the area of the larger region. Any polygonal region can be dissected into triangles. Draw a different polygonal region below and show how it can be divided into triangles.

- (d) Dissection can also be applied to volumes – we can break the large solid up into smaller solids. Highlight the three sections with three different colors.



DO NOW visual answer





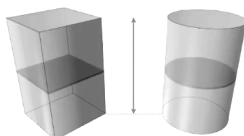
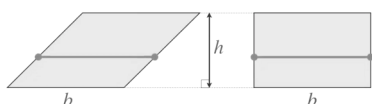
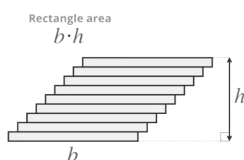
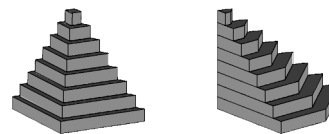
## (6) Cavalieri's Principle

**Cavalieri's Principle** -- Given two solids that are included between two parallel planes, if every plane parallel to the two planes intersects both solids in cross-sections of equal area, then the volumes of the two solids are equal.

Go to the websites below which demonstrate Cavalieri's Principle in 2D and 3D. Answer the questions for each part of the demonstration.

**2D:** <https://schoolyourself.org/learn/geometry/cavalieri-2d>

**3D:** <https://schoolyourself.org/learn/geometry/cavalieri-3d>



### 2D

- (1) The area of a rectangle is found by \_\_\_\_\_
- (2) The diagram at left shows that by slicing a rectangle and shifting each slice we get a shape that resembles a \_\_\_\_\_
- (3) The area of a parallelogram is found by \_\_\_\_\_

(4) Cavalieri's Principle for area states that \_\_\_\_\_

(5) The areas for the triangles at right are \_\_\_\_\_ because \_\_\_\_\_

(6) The areas for the shapes at left are \_\_\_\_\_ area as long as \_\_\_\_\_

### 3D

(7) Cavalieri's Principle for volume states that \_\_\_\_\_

(8) The volumes for the shapes at left are \_\_\_\_\_ because \_\_\_\_\_

(8)  
calculator
**Exit Ticket**

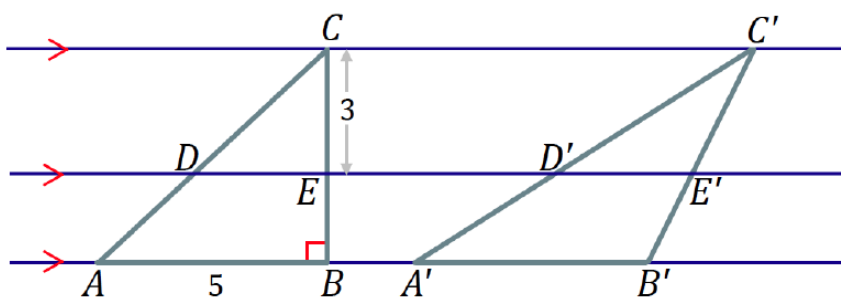
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 (9)  
calculator
**Homework**

Provide sufficient evidence for each response.

 (1)

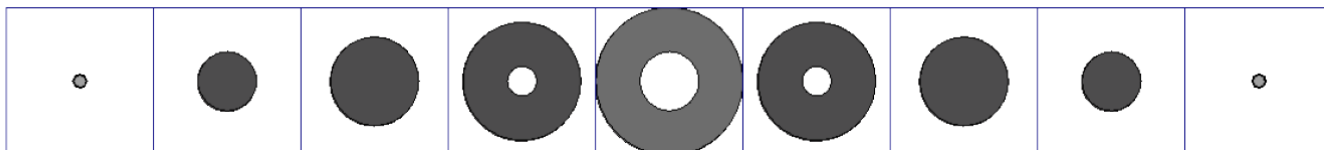
The following triangles have equal areas:  $\text{Area}(\triangle ABC) = \text{Area}(\triangle A'B'C') = 15 \text{ units}^2$ . The distance between  $\overline{DE}$  and  $\overline{CC'}$  is 3. Find the lengths  $\overline{DE}$  and  $\overline{D'E'}$ .

 (2)

Joey says that if two figures have the same height and the same area, then their cross-sectional lengths at each height will be the same. Give an example to show that Joey's theory is incorrect.

 (3)

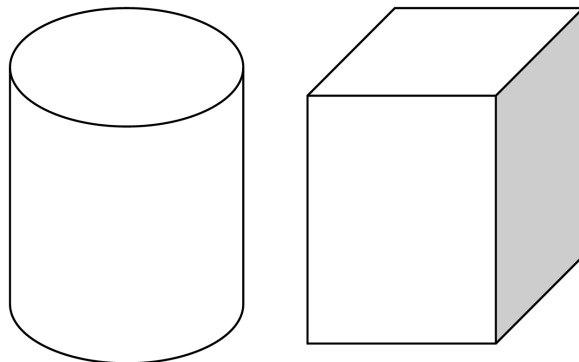
Horizontal slices of a solid are shown at various levels arranged from highest to lowest. What could the solid be?



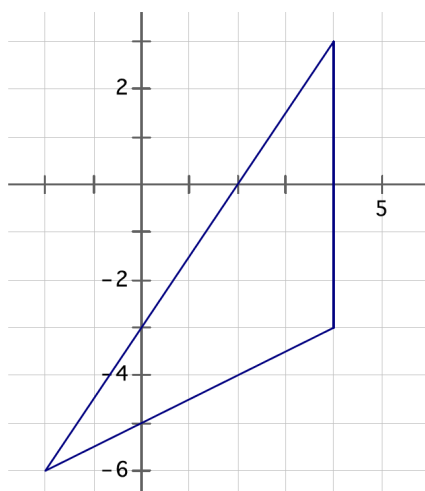
(6) **Cavalieri's Principle**  
internet

(4)

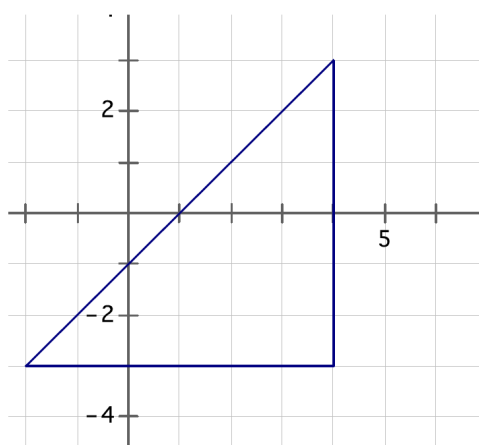
Morgan tells you that Cavalieri's principle cannot apply to the cylinders shown below because their bases are different. Do you agree or disagree? Explain.



(5) Use the principle of parallel slices to show that the two triangles below both have the same area.

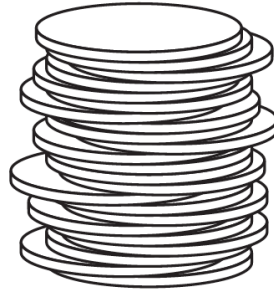
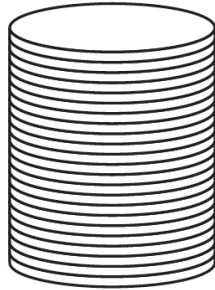


**Figure 2**



**Figure 3**

(1) Two stacks of 23 quarters each are shown below. One stack forms a cylinder but the other stack does not form a cylinder. Use Cavalieri's principle to explain why the volumes of these two stacks of quarters are equal.



DO NOW Name \_\_\_\_\_ Date \_\_\_\_\_ Per \_\_\_\_\_

9.1

(1) Which shape below has the most area?

